```
m = 1 / 100; Rmax = 300;
```

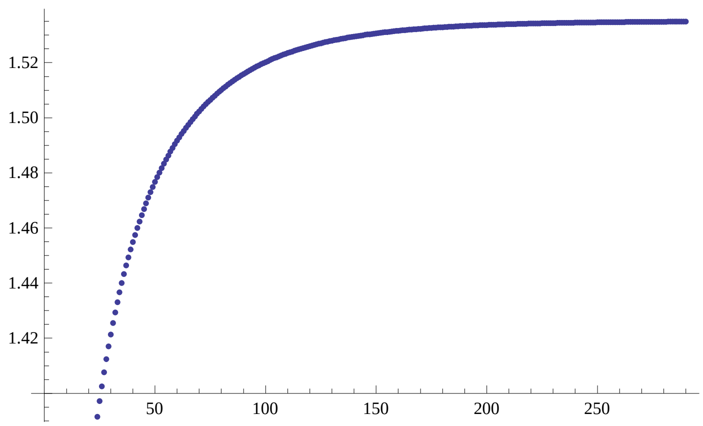
This takes a minute :

```
f = Table[NIntegrate[Exp[I x i] / 4 / Pi / Sqrt[m^2 + 2 (1 - Cos[x])], {x, -Pi, Pi}],
  {i, 0, Rmax}]; g = Table[
  NIntegrate[Exp[I x i] / 4 / Pi * Sqrt[m^2 + 2 (1 - Cos[x])], {x, -Pi, Pi}], {i, 0, Rmax}];

entropy[r_] := Module[{X, P, c}, X = Table[f[[Abs[i - j] + 1]], {i, 1, r}, {j, 1, r}];
  P = Table[g[[Abs[i - j] + 1]], {i, 1, r}, {j, 1, r}]; c = Sqrt[Eigenvalues[X.P]];
  Re[(c + 1 / 2).Log[c + 1 / 2] - (c - 1 / 2).Log[c - 1 / 2 + 10^(-10)]]]

ta = Table[{R, entropy[R]}, {R, 1, 290}];

ListPlot[ta]
```



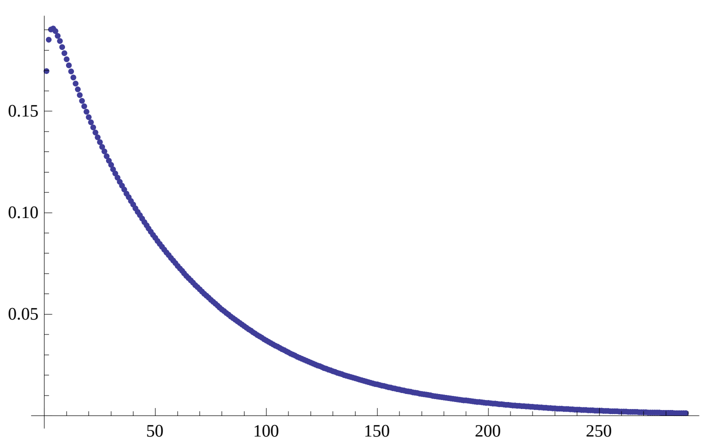
the saturation constant is approximately

```
ta[[290, 2]]
```

1.53491

```
cfunction =
  Table[{ta[[i, 1]], ta[[i, 1]] (ta[[i + 1, 2]] - ta[[i, 2]])}, {i, 1, Length[ta] - 1}];

ListPlot[cfunction]
```



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Now i try with a larger mass to see the mass dependence on the saturation constant

```
m = 1 / 10; Rmax = 40;
```

```
f = Table[NIntegrate[Exp[I x i] / 4 / Pi / Sqrt[m^2 + 2 (1 - Cos[x])], {x, -Pi, Pi}],
   {i, 0, Rmax}]; g = Table[
   NIntegrate[Exp[I x i] / 4 / Pi * Sqrt[m^2 + 2 (1 - Cos[x])], {x, -Pi, Pi}], {i, 0, Rmax}];
```

```
entropy[40]
```

```
0.770144
```

then fitting , it gives the coefficient - 1 / 3 in front of the log of the mass

```
Fit[{{1 / 10, 0.7701438146581794`}, {1 / 100, 1.534905060400002`}}, {1, Log[y]}, y]
```

```
0.00538257 - 0.332132 Log[y]
```

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

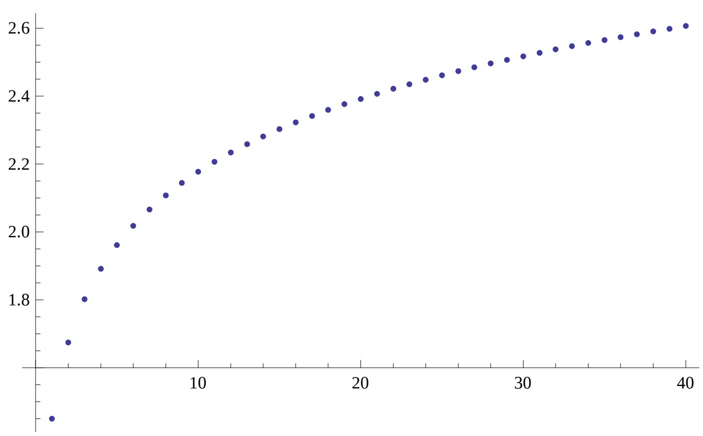
Now i try with a very small mass to see how the 1 / 3 Log[r] for the entropy is approached. I have to impruve numerical working precision a little bit

```
m = 10^(-10); Rmax = 40;
```

```
f = Table[NIntegrate[Exp[I x i] / 4 / Pi / Sqrt[m^2 + 2 (1 - Cos[x])], {x, -Pi, Pi},
    WorkingPrecision → 20, PrecisionGoal → 10, MaxRecursion → 20], {i, 0, Rmax}];
g = Table[NIntegrate[Exp[I x i] / 4 / Pi * Sqrt[m^2 + 2 (1 - Cos[x])], {x, -Pi, Pi},
    WorkingPrecision → 20, PrecisionGoal → 10, MaxRecursion → 20], {i, 0, Rmax}];
```

```
ta = Table[{R, entropy[R]}, {R, 1, 40}];
```

```
ListPlot[ta]
```




```
Fit[ta, {1, Log[y]}, y]
```

```
1.45841527462934 + 0.31161045821781 Log[y]
```

closer, but still not 0.3333 !